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Research article

Modeling of Nigerian Peak Petroleum Resources Depletion using Turbulent Flow Regime with Dead Time

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Abstract

Seven simple physical non-linear, mathematical models for petroleum resource depletion for Nigeria were developed by varying the input functions of the laplace domain of the models. The models were validated with data collected from the Department of Petroleum Resources (DPR), Ministry of Petroleum Resources, Nigeria, using MATLAB 7.9 software. As a control, these models were compared with an existing model by Hubert and found to be generally better. Two best models (III and V), with R^2 of 99.696% (oil), 99.680% (gas) and R^2 of 99.695% (oil), 99.641% (gas) respectively compared to Hubbert's model with R^2 of 98.67% (oil), 99.26% (gas), were selected out of seven (VII) models. The plots of the derivatives of the models which gave the annual production profile were also used to determine the peak and exhaustion periods for both oil and gas. The results were within range 2062 - 2121AD and 748982AD (infinity) respectively for oil production peak and exhaustion, and within the value 2782AD and 13853AD (infinity) respectively for gas production peak and exhaustion from 1957AD. The oil and gas ultimate recovery reserves were subsequently determined at 641.3TB and 5729Qscf respectively. Hence, even though the oil will peak before gas, gas will exhaust before oil. However, this can be due to the slight lower R^2 of gas as compared to that of the oil.

Keywords: Dead time, modeling, laplace domain, peak petroleum, turbulent.

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I. Background of the Study

The oil boom of early 1970s led Nigeria to neglect its strong agricultural and light manufacturing bases in favour of dependence on crude oil. She joined the Organization of Petroleum Exporting Countries (OPEC) in July 1971 as the eleventh member [2], [7]. Thereafter, the first huge increases in the price of crude oil in 1973 – 1974, due to Arab countries oil embargo yielded monumental financial benefits to the countries. Subsequently crude oil continued to play predominant and more strategic roles in the economy. An impression was therefore imprinted on the minds of nearly all Nigerians that no other area of our natural resources deserved to be further developed and nurtured except oil. The farm which was the main source of revenue prior to the discovery of petroleum was neglected as Nigerians found easier and less laborious means of earnings [13]. Youths migrated to urban centers, thus depleting the rural workforce. Consequently, production of both cash and staple crops fell drastically while the demand on the limited amenities and commodities out – stripped the rate of supply. This led to inflation which made none sense of their earnings [1], [2]. In 1973, the Middle East war broke, this aided Nigeria in making more money from their petroleum production. With the nation's desire for rapid development, our rising bills for imported food and industrial machinery, Nigeria became more and more dependent on oil. The question at that time was not how to get money, but how and on what to spend it; whether to spend on people – oriented projects or on gigantic white elephant scheme [12], [3].

Oil is a wasting asset, it cannot be replenished. Between 1974 and 1978, production remained stable on an average of 2.05 million barrels per day. Because this is non – replenishable, it means depletion [13], [28]. Then came the oil glut and suddenly, Nigerians realized that they had left undone essential things which they ought to have done. Global inflation was running into over 400 percent with the devaluation of our currency while the high rate of smuggling of our products, especially petroleum products, out of the country gained sway. Unemployment became a common occurrence [27]. The reserves held by a country constitute a strong determinant in assessment of the economic potentials of the country and rate of investment in oil and gas. Reserve is the much oil known to be producible, within a known time, with known techniques, at known cost and in known fields. It is not just the much oil in the rock in the earth. This is because the oil is divided into conventional and non – conventional oil [26], [29]. However, new discoveries, both onshore and offshore, do not mean new petroleum formation in the earth, it simply means either that another small 'old' reserve has been detected or that the 'former' large reserve has another place where it is closest to the surface of the earth. Many Nigerians may not know that their petroleum recoverable reserves are seriously depleting, even though we know that if a pool is always being scooped away everyday without replenishment, depletion and eventual exhaustion inevitably follows. The amount of oil and gas we get from the ground is about to reach its peak and starts coming down forever, no matter the ingenuity or determination to make it rise [31].

Following a global recession on world petroleum reserves, scholars like Deffeyes [9], [10]; Laherrere [22] – [24]; Cambell [6], Hubbert [18] – [20]; Delaney [11], Davey [8], Walsh [30] etc, have in one time or another attempted to alert the world of the danger that looms in the future fossil energy reserves, as this form of energy depletes exhaustibly. Strangely enough, Nigeria has no data record of her petroleum depletion scenario. Since the USA petroleum peaked in 1971, she has embarked on serious Middle East oil politics under the guise of exchanging –

terrorism – for – democracy wars on Afghanistan, Iraq, Kuwait, perhaps Iran, and friend of Saudi Arabia, Kuwait and who else! [12], [14]. Nigeria economy as at 2005 was about 90% petroleum – based to the detriment of other sectors because of false assumption that petroleum would be forever and that its depletion theory was unreal. This dependence is yet to change considering the present days' budget proposals. However, contrary to this erroneous view, Nigeria petroleum today is fast depleting at the rate to be determined in this research work. Just as many nations' oil reserves had peaked, some are still peaking and coming down like bumps of plateau. The problem is to find when Nigeria petroleum will peak or had peaked in the past years so that we can monitor the downward bumps of plateau to exhaustion. It is very important to have depletion models of Nigeria petroleum reserves so as to monitor the depletion profiles and to predict the dates of peaking and exhaustion. Since it is the mainstay of the country's economy, this knowledge will help the country quickly diversify the mode of revenue drive and save the nation from the shock of depletion crisis catching up with the mono – economy. This way, Nigerians will know that living in fantasy is not a good dream; that they have to face the reality – Nigerians oil, and indeed the world's is not limitless.

The broad aim of the study is to establish the model of the Nigerian petroleum depletion profile containing peaking and exhaustion dates. This study will do a detailed analysis on the depletion profiles especially at the peaks, plateaus, the slow upwards and fast downward bumps, and the exhaustion points. It is true that where the oil stops the gas continues but for how long! Since oil formation gives rise to gas formation or vise verse, the exhaustion of one leads to the eventual exhaustion of the other with time. Thus this study is not limited to just crude oil; natural gas depletion is also considered. This work covers only the conventional petroleum, both onshore and offshore reserves. It takes no account for non conventional oil since its production is yet to begin in Nigeria. Also, solid fossil fuels (coal and wood) are precluded in the study but allusions could still be extended to them. They are part of the fossilization process of these fuels.

II. Model Development

Model is a miniature representation of something, a pattern of something to be made, an example for imitation, or emulation, a description or analogy used to help visualize something that may or may not be directly observed, a system of postulates, data and inferences presented as a mathematical description of an entity or state of affairs [17]. Modeling is the concise description of the total variation in one quantity by partitioning it onto a deterministic component given by a mathematical function of one or more other quantities plus a random component that follows a particular probability. It is therefore the mathematical representation of physical system [4].

Assumptions

1. Assuming a composite underground crude petroleum reservoir among the rocks with a mean inflow and a mean outflow, where the outflow is the petroleum production as shown in the Fig. 1. This composite reservoir is assumed to contain all the Nigerian reserves and all the Nigerian crude is produced from it.
2. Production flow is turbulent i.e. $F_O = k\sqrt{v}$, but for weir flow, $F_O = kv^n$.
3. Constant density flow i.e. $\rho_i = \rho_o = \rho_*$

4. The Hubert's concept.

According to [19], data suitable for use in the prediction of oil and gas production peak and exhaustion are the cumulative production data generated from the annual production data. Such cumulative data must satisfy the following conditions:

i. Its plot (ogive) must be S – shaped (Sigmoidal).
ii. If the ogive is doubted, an S – shaped mathematical profile (cumulative model) is superimposed on the plotted curve to determine the percentage fitness (within the range 95% < $R^2 \leq$ 100%).
iii. The derivative of the model used on the ogive must generate a dumbbell profile which is the annual production time history.
iv. The ultimate recovery of the country's conventional petroleum must be determined at the asymptote of the resulting sigmoidal profile.
v. The country's petroleum production must peak at half the value of the ultimate value. This corresponds to half the value of the ultimate as read off on the y – axis, i.e. $t_{pk} = t_{\frac{1}{2}u}$.

This Hubbert's approach has been accepted to be explanatory to the bell curve, which was used by some Hubbert – modelers like: Campbell [5], Laherrere [21], etc. but as critized by some anti – Hubbert modelers like Lynch [25], etc., not all sigmoidal cumulative models give fused peak ($t_{pk} = t_{\frac{1}{2}u}$) as in logistic (or Verhust/Hubbert model). Instead, there are non – Hubbert sigmoidal cumulative models that give non – fused (range peaks, i.e. $t_{pk} \neq t_{\frac{1}{2}u}$). But [16] revealed that only models whose solutions are obtainable through the Bernoulli's technique obey $t_{pk} = t_{\frac{1}{2}u}$ and are known as logistics. Others have their peaks lying between $t_{pk} < t < t_{\frac{1}{2}u}$.

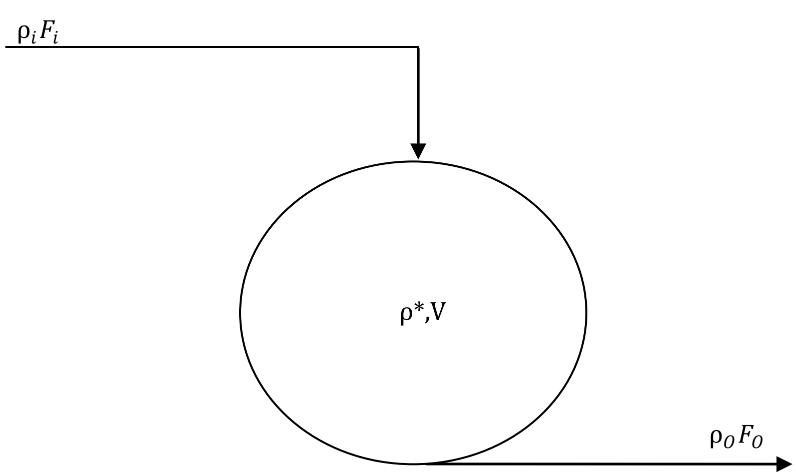

Figure 1: Composite Petroleum Reservoir

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Taking mass balance of the composite reservoir above:

Mass flow in – mass flow out = Rate of accumulation of mass in the reservoir.

i.e. $\rho_i F_i - \rho_o F_o = \frac{d}{dt}(\rho * V)$ ---------------- (1)

From the 2nd and 3rd assumptions, eqn. (1) becomes

$F_i - k\sqrt{V} = \frac{dv}{dt}$ ---------------- (2)

Linearizing $F_o = k\sqrt{V}$ gives

$$F_o = F_{(V)} = F_{(\bar{\nabla})} + (\frac{\partial F}{\partial V})_{(\bar{\nabla})}(V - \bar{\nabla}) + \frac{\partial^2 F}{\partial V^2}_{(\bar{\nabla})}\frac{(V-\bar{\nabla})^2}{2!} + \ldots$$

Truncating after second term to have;

$$F_o = F_{(V)} \approx F_{(\bar{\nabla})} + (\frac{1}{2} k\, \bar{\nabla}^{-\frac{1}{2}}))_{(\bar{\nabla})}(V - \bar{\nabla})$$

$$\approx k\sqrt{\bar{\nabla}} + \frac{K}{2}\frac{V}{\sqrt{\bar{\nabla}}} - \frac{K}{2}\sqrt{\bar{\nabla}}$$ ---------------- (3)

Substituting (3) into (2) after rearranging yields

$$\frac{dv}{dt} + \frac{K}{2}\frac{V}{\sqrt{\bar{\nabla}}} + \frac{K}{2}\sqrt{\bar{\nabla}} = F_i$$

If $\tau_p = 2\frac{\sqrt{\bar{\nabla}}}{k}$

Then, $\tau_p \frac{dv}{dt} + V = \tau_p F_i - \bar{\nabla}$ ---------------- (4)

Taking the laplace transform of equation (4) gives

$$\tau_p\left(SV_{(s)} - V_o\right) + V_{(s)} = \tau_p F_{i(s)} - \frac{\bar{\nabla}}{S}$$

$$S\left(S\tau_p + 1\right) V_{(s)} = S\tau_p \left(F_{i(s)} + V_o\right) - \bar{\nabla}$$

$$V_{(s)} = \frac{S\left(F_{i(s)} + V_o\right) - \frac{\bar{\nabla}}{\tau_p}}{S(S + \frac{1}{\tau_p})}$$ ---------------- (5a)

Applying dead time on the mean process using Pade's approximation;

i.e. $e^{-Ds} = \frac{e^{-\frac{Ds}{2}}}{e^{\frac{Ds}{2}}} \approx \frac{1-\frac{Ds}{2}}{1+\frac{Ds}{2}}$ (truncating after second term)

Inserting into (5a) yields

$$V_{(s)} = V_{(s)}e^{-Ds} = \frac{1-\frac{Ds}{2}}{1+\frac{Ds}{2}} \cdot \frac{S\left(F_{i(s)} + V_o\right) - \frac{\bar{\nabla}}{\tau}}{S(S+\frac{1}{\tau})}$$

$$V_{(s)} = \frac{2}{D} \cdot \frac{S\left(1-\frac{Ds}{2}\right)F_{i(s)} + V_o S\left(1-\frac{Ds}{2}\right) - \frac{\bar{\nabla}}{\tau}(1-\frac{Ds}{2})}{S\left(S+\frac{2}{D}\right)(S+\frac{1}{\tau})} \qquad \text{--------------- (5b)}$$

The input function $F_{i(s)}$ in (5b) will continue to be varied to see which model(s) gives the best fit of the raw data, from their statistical goodness of fit.

For zero input, $F_{i(s)} = 0$

$$V_{(s)} = \frac{2}{D} \cdot \frac{V_o S\left(1-\frac{Ds}{2}\right) - \frac{\bar{\nabla}}{\tau}(1-\frac{Ds}{2})}{S\left(S+\frac{2}{D}\right)(S+\frac{1}{\tau})} \equiv \frac{2}{D}\left(\frac{A}{S} + \frac{B}{S+\frac{2}{D}} + \frac{C}{S+\frac{1}{\tau}}\right)$$

$$V_{(t)1} = \frac{2(2\tau V_o + D\bar{\nabla})}{D-2\tau}\, e^{-\frac{2t}{D}} - \bar{\nabla} - \frac{(2\tau+D)(V_o+\bar{\nabla})}{D-2\tau}\, e^{-\frac{t}{\tau}} \qquad \text{--------------- (6a)}$$

For unit impulse, $F_{i(s)} = k$

$$V_{(s)} = \frac{2}{D} \cdot \frac{ks\left(1-\frac{Ds}{2}\right) + V_o S\left(1-\frac{Ds}{2}\right) - \frac{\bar{\nabla}}{\tau}(1-\frac{Ds}{2})}{S\left(S+\frac{2}{D}\right)(S+\frac{1}{\tau})} \equiv \frac{2}{D}\left(\frac{A}{S} + \frac{B}{S+\frac{2}{D}} + \frac{C}{S+\frac{1}{\tau}}\right)$$

$$V_{(t)2} = \frac{2(2k\tau + 2\tau V_o + D\bar{\nabla})}{D-2\tau}\, e^{-\frac{2t}{D}} - \bar{\nabla} - \frac{(2\tau+D)(V_o+\bar{\nabla})}{D-2\tau}\, e^{-\frac{t}{\tau}} \qquad \text{---------------- (7a)}$$

For unit step, $F_{i(s)} = \frac{k}{s}$

$$V_{(t)3} = k\tau - \bar{\nabla} + \frac{2(kD\tau + 2\tau V_o + D\bar{\nabla})}{2\tau - D}\, e^{-\frac{2t}{D}} - \frac{(2\tau+D)(k\tau - V_o - \bar{\nabla})}{2\tau - D}\, e^{-\frac{t}{\tau}} \quad \text{----- (8a)}$$

For ramp, $F_{i(s)} = \frac{k}{s^2}$

$$V_{(t)4} = k\tau t - \left(\frac{k\tau(D^2 - 2\tau^2 - D\tau)}{D-2\tau} + \bar{\nabla}\right) + \frac{(kD^2\tau + 4\tau V_o + 2D\bar{\nabla})}{D-2\tau}\, e^{-\frac{2t}{D}} - \frac{(2\tau+D)(k\tau^2 + V_o + \bar{\nabla})}{D-2\tau}\, e^{-\frac{t}{\tau}} \qquad \text{---------------- (9a)}$$

For unit pulse, $F_{i(s)} = \frac{k}{A}\frac{1-e^{-AS}}{S} \approx \frac{k}{A}.\frac{1-(1-SA+\frac{(SA)^2}{2})}{S} \approx k(1-\frac{AS}{2})$

$$V_{(t)5} = \frac{4k\tau(D+A)+D(2\tau V_o+D\bar{\nabla})}{D(D-2\tau)}e^{-\frac{2t}{D}} - \bar{\nabla} - \frac{(2\tau+D)(2k\tau+kA+2\tau V_o+2\tau\bar{\nabla})}{2\tau(D-2\tau)}e^{-\frac{t}{\tau}} \quad \text{--------------- (10a)}$$

If D = A, i.e. when dead time is equivalent to the duration of unit pulse

$$V_{(t)5i} = \frac{2(4k\tau+2\tau V_o+D\bar{\nabla})}{(D-2\tau)}e^{-\frac{2t}{D}} - \bar{\nabla} - \frac{(2\tau+D)(2\tau(k+V_o+\bar{\nabla})+kD)}{2\tau(D-2\tau)}e^{-\frac{t}{\tau}} \quad \text{-------------- (11a)}$$

If input is an exponential, $F_{i(s)} = \frac{k}{S+k_1}$

$$V_{(t)6} = 2\left\{\frac{2kD\tau^2}{(D-2\tau)^2} + \frac{(2V_o\tau+D\bar{\nabla})}{(D-2\tau)}\right\}e^{-\frac{2t}{D}} - \bar{\nabla} - \frac{k(2\tau+D)}{(2\tau-D)}t + \left(\bar{\nabla} - V_o - \frac{2(2V_o\tau+D\bar{\nabla})}{(D-2\tau)} - \frac{4kD\tau^2}{(D-2\tau)^2}\right)e^{-\frac{t}{\tau}} \quad \text{-------------- (12a)}$$

The corresponding annual profiles are given as:

$$P_{(t)1} = \frac{(2\tau+D)(V_o+\bar{\nabla})}{\tau(D-2\tau)}e^{-\frac{t}{\tau}} - \frac{4(2\tau V_o+D\bar{\nabla})}{D(D-2\tau)}e^{-\frac{2t}{D}} \quad \text{--------------- (6b)}$$

$$T_{pk1} = \frac{D\tau}{2\tau-D}\ln\frac{4\tau}{D}\frac{(2\tau V_o+D\bar{\nabla})}{(2\tau+D)(V_o+\bar{\nabla})} \quad \text{--------------- (6c)}$$

$$P_{(t)2} = \frac{(2\tau+D)(k+V_o+\bar{\nabla})}{\tau(D-2\tau)}e^{-\frac{t}{\tau}} - \frac{4(2k\tau+2\tau V_o+D\bar{\nabla})}{D(D-2\tau)}e^{-\frac{2t}{D}} \quad \text{--------------- (7b)}$$

$$T_{pk2} = \frac{D\tau}{2\tau-D}\ln\frac{4\tau}{D}\frac{(2k\tau+2\tau V_o+D\bar{\nabla})}{(2\tau+D)(k+V_o+\bar{\nabla})} \quad \text{--------------- (7c)}$$

$$P_{(t)3} = \frac{(2\tau+D)(k\tau-V_o-\bar{\nabla})}{\tau(D-2\tau)}e^{-\frac{t}{D}} - \frac{4(kD\tau-2\tau V_o-D\bar{\nabla})}{D(2\tau-D)}e^{-\frac{2t}{D}} \quad \text{--------------- (8b)}$$

$$T_{pk3} = \frac{D\tau}{2\tau-D}\ln\frac{4\tau}{D}\frac{(kD\tau-2\tau V_o-D\bar{\nabla})}{(2\tau+D)(k\tau-V_o-\bar{\nabla})} \quad \text{--------------- (8c)}$$

$$P_{(t)5} = \frac{(2\tau+D)(2k\tau+kA+2\tau V_o+2\tau\bar{\nabla})}{2\tau^2(D-2\tau)}e^{-\frac{t}{\tau}} - \frac{8k\tau(D+A)+2D(2\tau V_o+D\bar{\nabla})}{D^2(D-2\tau)}e^{-\frac{2t}{D}} \quad \text{-------------- (10b)}$$

$$T_{pk5} = \frac{D\tau}{2\tau-D}\ln 2\left(\frac{\tau}{D}\right)^2\frac{8k\tau(D+A)+2D(2\tau V_o+D\bar{\nabla})}{(2\tau+D)(2k\tau+kA+2\tau V_o+2\tau\bar{\nabla})} \quad \text{-------------- (10c)}$$

$$P_{(t)6} = \frac{(2\tau+D)(2\tau(k+V_o+\bar{\nabla})+kD}{2\tau^2(D-2\tau)}e^{-\frac{t}{\tau}} - \frac{4(4k\tau+2\tau V_o+D\bar{\nabla})}{D(D-2\tau)}e^{-\frac{2t}{D}} \quad \text{-------------- (11b)}$$

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$$T_{pk6} = \frac{D\tau}{2\tau - D} ln \frac{8\tau^2}{D} \frac{(4k\tau + 2\tau V_o + D\bar{v})}{(2\tau + D)(2\tau(k + V_o + \bar{v}) + kD}$$ -------------- (11c)

Table 1: Summary of the Developed Models Used in this Work

MODEL NO.	INPUT FUNCTION	CUMULATIVE PRODUCTION: $V_{(t)}$	ANNUAL PRODUCTION: $P_{(t)} = \frac{dv}{dt}$	PEAK TIME PRODUCTION: T_{pk} at $\frac{d^2v}{dt^2} = \frac{dp}{dt} = 0$
I	0	$\frac{2(2\tau V_o + D\bar{v})}{D - 2\tau} e^{-\frac{2t}{D}} - \bar{v} - \frac{(2\tau + D)(V_o + \bar{v})}{D - 2\tau} e^{-\frac{t}{\tau}}$	$\frac{(2\tau + D)(V_o + \bar{v})}{\tau(D - 2\tau)} e^{-\frac{t}{\tau}} - \frac{4(2\tau V_o + D\bar{v})}{D(D - 2\tau)} e^{-\frac{2t}{D}}$	$\frac{D\tau}{2\tau - D} ln \frac{4\tau}{D} \frac{(2\tau V_o + D\bar{v})}{(2\tau + D)(V_o + \bar{v})}$
II	Unit Impulse, K	$\frac{2(2k\tau + 2\tau V_o + D\bar{v})}{D - 2\tau} e^{-\frac{2t}{D}} - \bar{v} - \frac{(2\tau + D)(V_o + \bar{v})}{D - 2\tau} e^{-\frac{t}{\tau}}$	$\frac{(2\tau + D)(k + V_o + \bar{v})}{\tau(D - 2\tau)} e^{-\frac{t}{\tau}} - \frac{4(2k\tau + 2\tau V_o + D\bar{v})}{D(D - 2\tau)} e^{-\frac{2t}{D}}$	$\frac{D\tau}{2\tau - D} ln \frac{4\tau}{D} \frac{(2k\tau + 2\tau V_o + D\bar{v})}{(2\tau + D)(k + V_o + \bar{v})}$
III	Unit step, $\frac{k}{s}$	$k\tau - \bar{v} + \frac{2(kD\tau + 2\tau V_o + D\bar{v})}{2\tau - D} e^{-\frac{2t}{D}} - \frac{(2\tau + D)(k\tau - V_o - \bar{v})}{2\tau - D} e^{-\frac{t}{\tau}}$	$\frac{(2\tau + D)(k\tau - V_o - \bar{v})}{\tau(D - 2\tau)} e^{-\frac{t}{D}} - \frac{4(kD\tau - 2\tau V_o - D\bar{v})}{D(2\tau - D)} e^{-\frac{2t}{D}}$	$\frac{D\tau}{2\tau - D} ln \frac{4\tau}{D} \frac{(kD\tau - 2\tau V_o - D\bar{v})}{(2\tau + D)(k\tau - V_o - \bar{v})}$
IV	Ramp, $\frac{k}{s^2}$	$k\tau t - \left(\frac{k\tau(D^2 - 2\tau^2 - D\tau)}{D - 2\tau} + \bar{v}\right) + \frac{(kD^2\tau + 4\tau V_o + 2D\bar{v})}{D - 2\tau} e^{-\frac{2t}{D}} - \frac{(2\tau + D)(k\tau^2 + V_o + \bar{v})}{D - 2\tau} e^{-\frac{t}{\tau}}$	---	---
V	Unit Pulse, $k(1 - \frac{AS}{2})$	$\frac{4k\tau(D + A) + D(2\tau V_o + D\bar{v})}{D(D - 2\tau)} e^{-\frac{2t}{D}} - \bar{v} - \frac{(2\tau + D)(2k\tau + kA + 2\tau V_o + 2\tau\bar{v})}{2\tau(D - 2\tau)} e^{-\frac{t}{\tau}}$	$\frac{(2\tau + D)(2k\tau + kA + 2\tau V_o + 2\tau\bar{v})}{2\tau^2(D - 2\tau)} e^{-\frac{t}{\tau}} - \frac{8k\tau(D + A) + 2D(2\tau V_o + D\bar{v})}{D^2(D - 2\tau)} e^{-\frac{2t}{D}}$	$\frac{D\tau}{2\tau - D} ln\, 2(\frac{\tau}{D})^2 \frac{8k\tau(D + A) + 2D(2\tau V_o + D\bar{v})}{(2\tau + D)(2k\tau + kA + 2\tau V_o + 2\tau\bar{v})}$
VI	Unit Pulse, A = D, $k(1 - \frac{DS}{2})$	$\frac{2(4k\tau + 2\tau V_o + D\bar{v})}{(D - 2\tau)} e^{-\frac{2t}{D}} - \bar{v} - \frac{(2\tau + D)(2\tau(k + V_o + \bar{v}) + kD)}{2\tau(D - 2\tau)} e^{-\frac{t}{\tau}}$	$\frac{(2\tau + D)(2\tau(k + V_o + \bar{v}) + kD}{2\tau^2(D - 2\tau)} e^{-\frac{t}{\tau}} - \frac{4(4k\tau + 2\tau V_o + D\bar{v})}{D(D - 2\tau)} e^{-\frac{2t}{D}}$	$\frac{D\tau}{2\tau - D} ln \frac{8\tau^2}{D} \frac{(4k\tau + 2\tau V_o + D\bar{v})}{(2\tau + D)(2\tau(k + V_o + \bar{v}) + kD}$
VII	Exponential, $\frac{k}{s + k_1}$	$2\left\{\frac{2kD\tau^2}{(D - 2\tau)^2} + \frac{(2V_o\tau + D\bar{v})}{(D - 2\tau)}\right\} e^{-\frac{2t}{D}} - \bar{v} - \frac{k(2\tau + D)}{(2\tau - D)} t + \left(\bar{v} - V_o -\right.$ 22Voτ+D∇D−2τ− 4kDτ2D−2τ2e−tτ	---	---

III. Data Collection

The total production data for Nigerian crude petroleum (Oil and Gas) for all the oil companies operating in Nigeria from 1957 to 2007 were obtained from the Department of petroleum Resources (DPR), Ministry of Petroleum and Minerals, 7, Kofo Abayomi street, Victoria Island, Lagos.

A. Plots of Collected Data (Curve Fitting)

Models 6a - 12a, were respectively superimposed on the raw cumulative scatter diagram for oil and gas to ascertain the fitness of the sigmoidal profiles as declared Matlab 7.9 software. The first derivatives of these models (6b - 12 b) were also plotted with the differentiated data of the sigmoidal profiles to obtain dumb - bell profiles.

The ultimate production value, the period and $t_{\frac{1}{2}u}$ were read off from the sigmoidal profiles, while the peak and exhaustion values were read off from the corresponding bumb – bell profiles.

IV. Results

The results of superimposing models 6a – 12a and models 6b – 12b on cumulative and annual data scatter diagrams are shown to have the same shapes as in figures 2a, 3a and figures 2b, 3b respectively for oil and gas. Tables 2 and 3 are the model III parameter values and the entire model values respectively.

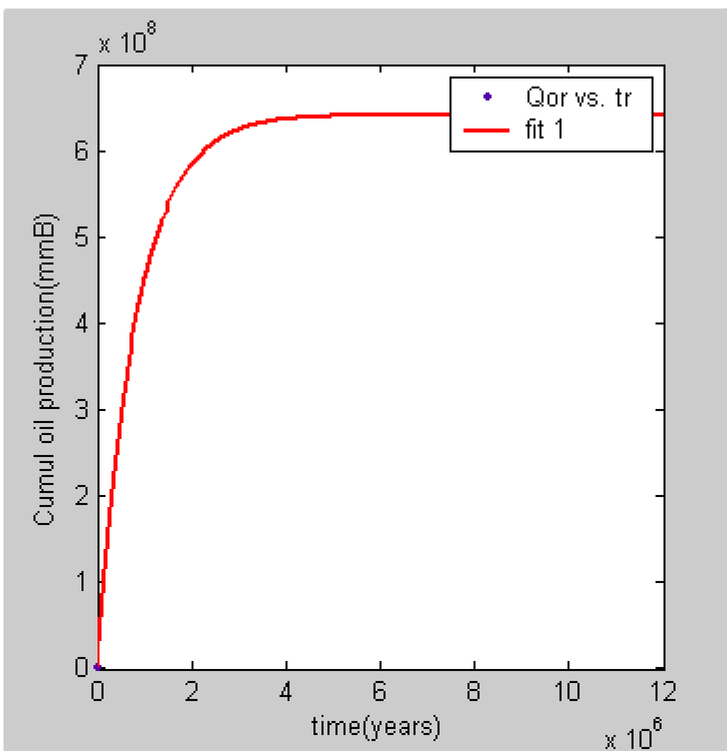

Figure 2a: Cumulative oil production versus time (R^2=0.99696); k/s

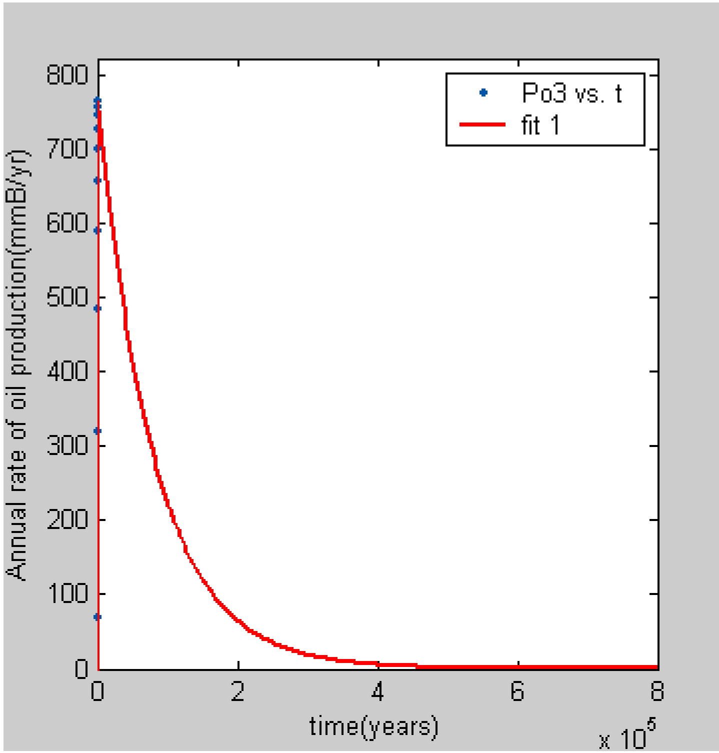

Figure 2b: Annual rate of oil production versus time k/s, D

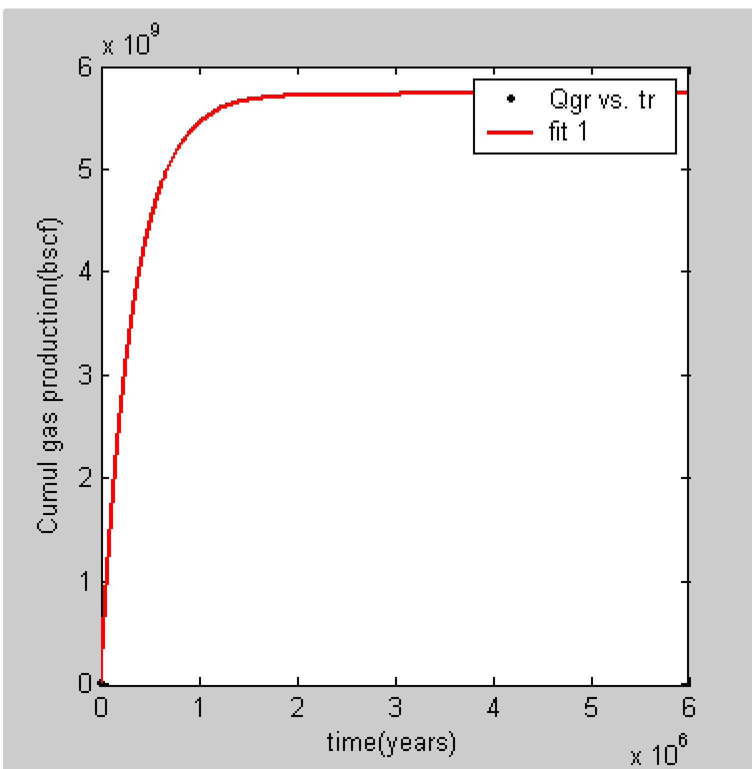

Figure 3a: Cumulative gas production versus time (R^2=0.99680); k/s

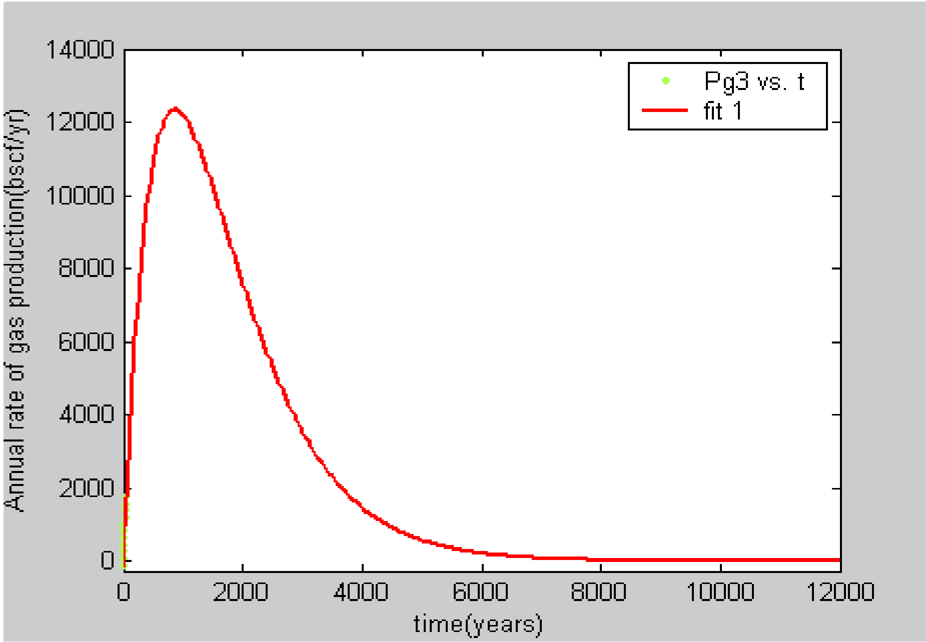

Figure 3b: Annual rate of gas production versus time k/s, D

Table 2: Coefficients and Goodness of fit of model III (Oil and Gas Production)

Oil; Coefs. (with 95% confidence bounds)		**Gas; Coefs. (with 95% confidence bounds)**	
D	= 22.82	D	= 897.2
U	= 1.622×10^4	U	= 1.634×10^6
$V_1 = \bar{v}$	= 1.302×10^4	$V_1 = \bar{v}$	= 1733
V_0	= 0.3416	V_0	= 4.766
$a = \tau$	= 8.219×10^5	$a = \tau$	= 3.267
K	= 0.06395	K	= 0.01604
U: f (1.14934×10^7)	= 6.41322×10^8	U: f (5.29981×10^6)	= 5.72868×10^9
$\boldsymbol{t_{\frac{1}{2}u}}$: f(569,829)	= 3.20661×10^8	$\boldsymbol{t_{\frac{1}{2}u}}$: f(226,918)	= 2.86434×10^9
$\boldsymbol{t_{pk}}$: f(104.748)	= 779.567	$\boldsymbol{t_{pk}}$: f(824.939)	= 12,329.7
$\boldsymbol{t_{ex}}$: f(747,025)	= 0.07274	$\boldsymbol{t_{ex}}$: f(11,896.3)	= 0.426032
Goodness of fit		**Goodness of fit**	
SSE	= 1.187×10^7	SSE	= 2.519×10^7
R^2	= 0.99696	R^2	= 0.9968
R^2 – Adj.	= 0.9966	R^2 – Adj.	= 0.9964
RMSE	= 513.7	RMSE	= 748.2

Table 3: Results of the Petroleum Production depletion models developed

Model No.	Input function	Petroleum	R^2	RMSE	Peak production Pm	Peak period (yrs) t_{pk} - $t_{\frac{1}{2}u}$	Ultimate Recovery U	Exhaustion period(yrs) t_{ex} - t_u	Remarks
I	Zero O	Oil	0.9939	706	-	- 9797	9.645×10^6mmB	- 194,932	Doesn't predict peak and exhaustion
		Gas	0.9723	2166	-	- 1885.6	2.961×10^6bscf	- 37,334	Doesn't predict peak and exhaustion
II	Unit impulse K	Oil	0.9941	716.4	692	55 – 29,460	2.946×10^7mmB	389,107 – 547,343	Satisfactory
		Gas	0.9734	214.6	1093.5	77 – 47,153	7.445×10^7bscf	531,669 – 935,484	Satisfactory
III	Unit step $\frac{k}{S}$	Oil	0.9969	513.7	779.6	105 – 569829	6.413×10^8mmB	747,025 – 11,493,400	Very good
		Gas	0.9968	748.2	12329.7	825 – 226,918	5.729×10^9bscf	11,896 – 5,299,810	Very good
IV	Unit Pulse $k(1-\frac{AS}{2})$	Oil	0.9969	525.6	768	76 – 907	1.0124×10^6mmB	- 14,924	No prediction of t_{ex} : 642 mmB
		Gas	0.9966	779.6	-	- 2259	3.355×10^7bscf	- 39,706	Doesn't predict peak and exhaustion
V	Unit Pulse A = D k(1 -$\frac{DS}{2}$)	Oil	0.99695	514.4	776	164 – 8,755,220	9.798×10^9mmB	116,948,000 – 188,899,000	Very good
		Gas	0.99641	788.1	10,759	1882 – 2118600	1.894×10^{10}bscf	2,949,140 – 39,041,700	Good
VI	Exponential $\frac{k}{S+k_1}$	Oil	0.99699	510.4	-	-	-	-	Does not predict
		Gas	0.99686	736.4	-	-	-	-	Does not predict
VII	Ramp $\frac{k}{s^2}$	Oil	0.99699	570.5	-	-	-	-	Does not predict
		Gas	0.9963	763	-	-	-	-	Does not predict
VIII	Hubbert	Oil	0.9862	1038	849.3	35 – 35.2	31,517.7	134.7 – 158.2	
		Gas	0.9926	1094	1605.2	45 – 45.3	65,998.5	144.2 – 183.2	

V. Discussion

A. General Discussion

From the remark column of Table 3, statements or phrases have been put describing the fitness of the models to the Nigerian petroleum production data. When the input function is zero, the model developed could not predict peak and exhaustion periods, nor could it predict peak production. It therefore means that there is always an input in Nigerian petroleum reserve.

When the input is a unit impulse, the model gave a low and satisfactory prediction comparatively. But when the input is unit step, the model gave very good prediction as seen from the R^2 (coefficient of correlation). It therefore means that Nigerian petroleum reserve shows a constant value of input with increasing time. However, when input function is unit pulse, the model either does not predict peak and exhaustion or predict wrong exhaustion period at annual production value of 642mmb instead of at zero (0). When the unit pulse input function is adjusted as seen in Model V, it gave a very good oil prediction and a good gas prediction i.e. when dead time is equivalent to the duration of unit pulse. The model does not predict at all when input function is either exponential or ramp. When compared with Hubbert (the control model), the Hubbert model gave fused peaks for oil and gas, since it is a logistic model.

It can be seen that the developed models, all gave none fused (range of peaks) as t_{pk} is not equal to $t_{\frac{1}{2}u}$ ($t_{pk} \neq t_{\frac{1}{2}u}$), in line with anti – Hubbert modeler, Lynch [24]. But, Hubbert model gave fused peak since it is a logistic growth model, as opined by Hubbert modelers: Campbell [5], [6] and Laherrere [21], [22].

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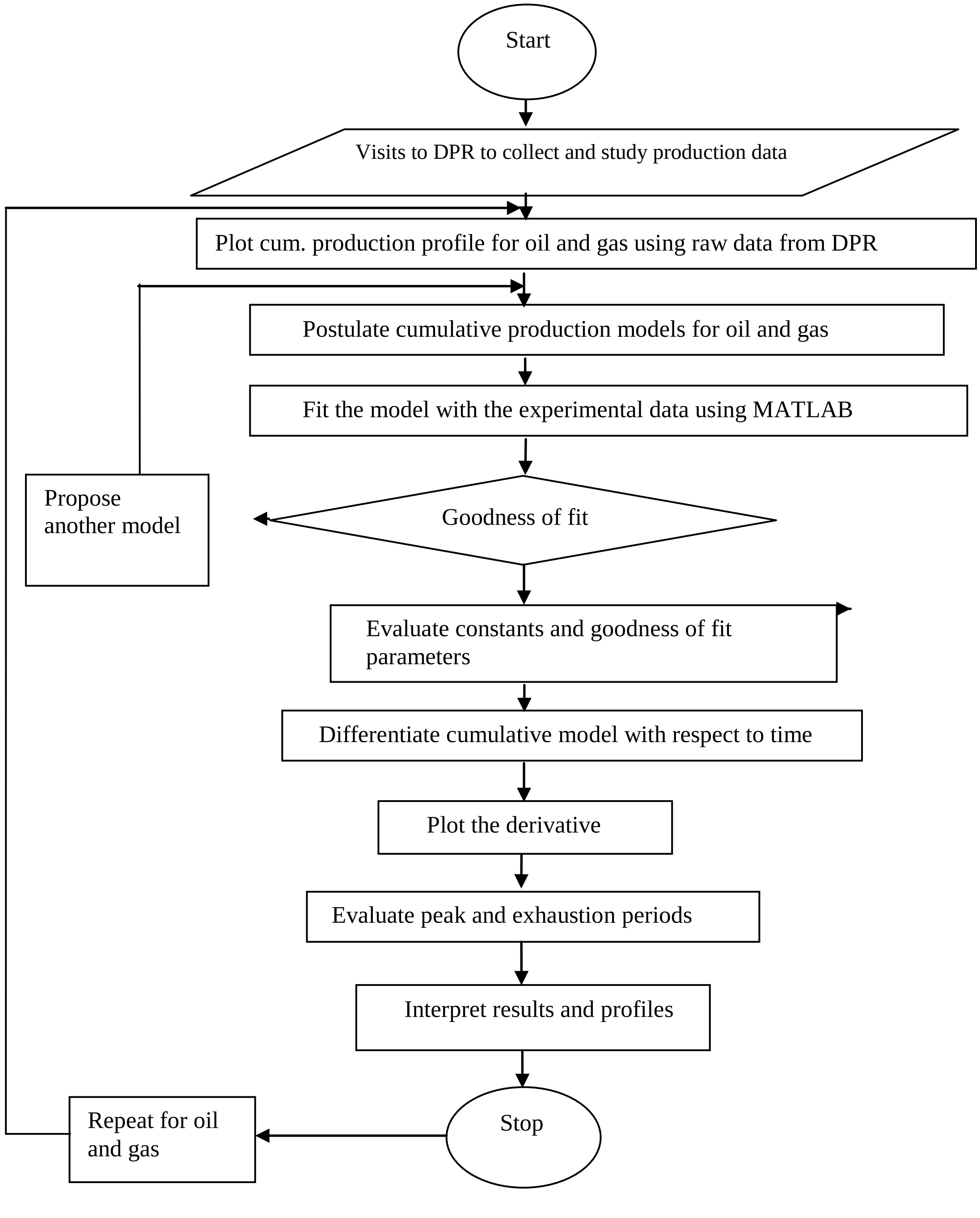

Figure 4: General Flowchart for methology/work plan for modeling of Prediction and Analysis of Nigeria's Crude Oil and Gas Resources Depletion.

B. The Best Models

Models 3 and 5 revealed themselves as the best models since apart from predicting well, their R^2 are highest for those that predicted well. But it is said that when the range of t_{pk} to $t_{\frac{1}{2}u}$ are in tens of centuries, t_{pk} is the correct peak. Therefore, in Models III and V, the oil will peak between 105 and 164 years from 1957, i.e. 2062 – 2121 AD. Having peak production of 778mmb, with ultimate production value of 6.413×10^8mmb (641,300BB = 641.3TB), the exhaustion period which is 747,025 years simply means infinite. For the gas, the peak production period is 825 years from 1957, which is 2782 AD with peak production 12,330bscf (approximately 12Tscf). The ultimate production is put at 5.729×10^9bscf (5.729 10^6Tscf = 5,729Qscf). With the exhaustion period put at 11,896 years meaning infinite again. The best models have predicted the Nigerian petroleum depletion very well, with R^2s for oil and gas as 0.99696 and 0.99680 respectively. The research showed that, contrary to public view that only gas will last for a long time, oil as well will last for a very long time. And, Nigeria can supply both oil and gas to the entire West Africa and gas to the entire Africa for a very long time, just as Russia supplies the entire Europe with gas through Ukraine.

Hubbert [18] used this approach to predict that American lower 48 States petroleum would peak in the nineteen seventies and it exactly did in 1971. Since then America has been going down their production plateau and augmenting their reserves supply with Saudi Arabian, Kuwaiti, Iraqi etc petroleum. Hubbert's principles of 1956 were applied in Nigeria case in determining the production peak and exhaustion values.

The Nigerian oil production is to peak between 2062 – 2121 AD, over 59 year's interval. Its exhaustion period, however, is elongated, tapering out slowly to 747,025 years (infinity), as the yield towards the end will be shrinking towards an uneconomic smallness (0.072MMB/a). The ultimate recovery will come to 641.3TB. It means that at almost infinity, conventional oil will economically and officially exhaust at an ultimate recovery of 641.3TB. That is, Nigeria oil will not exhaust for a very long time (747025 years from 1957AD). On the other hand, gas production will continue and peak 2782AD. Its exhaustion is elongated tapering out slowly to 11,896 years (infinity), though the yield towards the end will be shrinking towards uneconomic smallness (0.432bscf/a). The ultimate recovery will come to 5729Qscf. It means that Nigerian gas will not exhaust for a very long time (11896 years). In a lighter mood, Mikel de Nostradamus predicted 3797AD as the end of the world: these exhaustion periods are beyond this date.

C. Predicting Nigerian Petroleum Production on Hubbert Curve

The initial theory behind what is now known as the Hubbert curve was very simplistic. Hubbert was simply trying to estimate approximate resource levels, and for the lower – 48 U.S., he thought a bell – curve would be most appropriate form. It was only later that the Hubbert curve came to be seen as self - explanatory, that is, geology requires that production should follow such a curve. Indeed, for many years, Hubbert himself published no equations for deriving the curve, and it appears that he only used a rough estimation initially. In his 1956 paper, in fact, he noted that production often did not follow a bell curve. In later years, however, he seemed to have accepted the curve as explanatory [18]. Hubbert theorized that it is the rock (geology) and not technology that determines how much petroleum to let go from a reservoir. For a closed system, such as the U.S gas market, demand determines production, not geology.

This Hubbert approach that has been accepted to be explanatory to bell curve, that was used by some Hubbert – modelers like Campbell [5], Laharrere [21] etc, was also used for predicting Nigerian Petroleum production. But as criticized by some anti – Hubbert – Modelers like Lynch [25], not all sigmoidal cumulative models gave fused peak ($t_{pk} = t_{1/2u}$) as in logistic (or Verhulst/Hubbert model). This work revealed that and as did 18 out of 20 arrays of models developed in [17]. Instead, there were non – Hubbert sigmoidal cumulative models that gave non – fused (range) peaks ($t_{pk} \neq t_{1/2u}$). t_{pk} is the peak time of the bumb – bell profile produced by plotting annual production against time, while $t_{1/2u}$ is, also, a peak time (according to Hubbert – modelers) occurring at half the ultimate recovery, U, on a sigmoidal profile of the plot of cumulative production against time [6], [22] – [24].

VI. Conclusion

Simple physical models of petroleum resource depletion for Nigeria have been developed. The models are based on Nigerian's oil and gas production data between 1957 and 2007 (50years) collected from the Department of Petroleum Resources (DPR) of the Ministry of Petroleum Resources. The MATLAB software was employed in the determination of the model equation constants profiles and statistical goodness of fit. From the work, the following conclusions were obtained:

1. The peak and exhaustion periods for Nigerian oil and gas production are between 2062 – 2121AD and 748982AD (infinity) respectively for oil production and between the years 2782AD and 13853AD (infinity) for gas production respectively (see Table 3)
2. Gas production peak will occur roughly 720 years after those of oil (at 825 years), and earlier exhaustion of 735,129 years (infinity) before that of oil can occur (at 11896 years from 1957).
3. One model can be used effectively to predict best for both oil and gas (Model III).
4. Not all sigmoidal models are Hubbert type. In Hubbert sigmoidal models, the two production peaks fuses (merges) into a single peak, while, in non – Hubbert sigmoidal models the two production peaks form a range.
5. The production data from a particular country used in determining the values of the constants in the model gives its specific characteristics and different from all other countries. Consequently, a particular model is best for a particular country.

VII. Achievements of the Present Study

The achievements made from this study are so numerous that time and space can only permit mentioning of a few:

1. Development of simple physical Nigerian petroleum reserve depletion models with which the geologically – imposed peak production, ultimate recovery, and exhaustion periods of petroleum reserve phenomenon for oil and gas can be predicted with reasonable accuracy.
2. Determination of the production peak and exhaustion periods for oil and gas in Nigeria as well as her oil and gas reserves.

3. This study discovered and unveiled the high volume of oil and gas reserves of Nigeria comparable only to the quantity of Russia gas in the world; as Nigeria supplies most of the gas used in the entire Africa, her gas needs for decades.
4. It also shows that petroleum reserve depletion obeys the law of diminishing returns like other exhaustible natural resources.
5. It goes on to show that petroleum production peak, ultimate recovery and exhaustion periods, as determined by the geology of the soil, are calculable like other natural disaster occurrence dates e.g. earth tremors, volcanic eruptions, earth flooding, hurricane Catherina etc.
6. Nigerian Petroleum Industry will last for a very long time.

VIII. Recommendations

With the above findings, Nigeria still needs alternative or parallel projects though her petroleum resources will last for a very long foreseeable future. Parallel projects like embarking on intensive and extensive agriculture, solid mineral exploitation, tapping of solar energy etc will do. Furthermore, Nigeria should begin to study the technology of converting oil – fuelled industrial operations, vehicles, sea – moving and air – moving machines to gas – fuelled so that gas flaring can be converted to useful ventures. I recommend also the study of non – conventional petroleum reserves depletion, production peaking and exhaustion as a further study to be undertaken separately by someone else, since it is not covered in this research.

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